Title of the Research:

A study on the ideals of semigroups and ordered semigroups

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SYNOPSIS

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1 Introduction

In 1952, R.A Good and D.R Hughes[4] introduced the notion of bi-ideals of a semigroup. A special case of (m,n)-ideals was introduced by S.Lajos. Quasi ideals which was introduced by Otto Steinfield are generalizations of both left ideals and right ideals as well as a particular case of bi-ideals. the characterizations of regular and its subclasses like completely regular semigroups,clifford semigroups by quasi-ideals and bi-ideals was beautifully presented by S.Lajos.[2],[3].

2 Background

Kehayopulu introduced the notion of bi-ideals and quasi-ideals in an ordered semigroup and she developed the theory of regular, completely regular and t-simple ordered semigroups by their bi-ideals and quasi- ideals Later, these types of characterizations in ordered semigroups have been studied by Lee, Kang and others. This work is inspired by the works of S.Lajos. Clifford(left clifford) ordered semigroup is a subclass of ordered semigroup. The clifford(left clifford) ordered semigroup has been introduced in [1]. Hansda [3] have characterized clifford and left clifford ordered semigroups by their bi-ideals and quasi-ideals. The purpose of this paper is to characterize set of all bi-ideals $B(S)$ of a regular, intra-regular and Clifford ordered semigroup S.

3 Brief History of Ordered Semigroup:

An ordered semigroup is a partiality ordered set $(S, \cdot, \leq)$, and at the same time a semigroup $(S, \cdot)$ such that for all $a,\ b,\ x \in S$ $a \leq b$ implies $xa \leq xb$ and $ax \leq bx$. It is denoted by $(S, \cdot, \leq)$.

A band is a semigroup in which every element is an idempotent. An ordered semigroup S is called an band if the semigroup $(s, \cdot)$ is a band. A commutative band is called a semilattice. An ordered semigroup S is called $\mathcal{H}$-commutative if for every $a, b \in S$ there is $x \in S$ such that $ab \leq bxa$. An ordered semigroup S is called an regular ordered semigroup if for every $a \in S$, $a \in (aSa]$. For an immediate example of a regular ordered semigroup consider $(\mathbb{N}, \cdot, \leq)$. One can see that $(\mathbb{N}, \cdot)$ is not regular as an ordered semigroup, as for example $2 \leq 2 \cdot 2 \cdot 2$. An ordered semigroup S is called intra-regular semigroup if for every $a \in S$,$a \in (Sa^2S]$. An ordered semigroup S is called clifford if S is $\mathcal{H}$-commutative. Let I be a nonempty subset of an ordered semigroup S. I is a left ideal of S, if $SI \subseteq I$ and $(I] = I$. The right ideals are defined dually. A subset I of S is called a two-sided ideal or briefly an ideal of S if it is both a left and a right ideal of S. A subsemigroup B of S is bi-ideal of S if $BSB \subseteq B$ and $(B] = B$. Since intersection of any collection of ideals (left,right,bi)containing $a \in S$ which we denote by $I(a)$($L(a)$,$R(a)$,$B(a)$) respectively, and call as the principal ideal (left,right,bi) of S,respectively, generated by $a \in S$, defined as follows:

1. $L(a) = (a \bigcup Sa]$
2. $R(a) = (a \bigcup aS]$
3. $I(a) = (a \bigcup Sa \bigcup aS \bigcup SaS]$
4. $B(a) = (a \bigcup a^2 \bigcup aSa]$.

4 Scope of Work:

An ordered semigroup S is called $\mathcal{H}$-commutative if for every $a, b \in S$ there is $x \in S$ such that $ab \leq bxa$ and a regular ordered semigroup S is called clifford if S is $\mathcal{H}$-commutative. Here we have characterized set of all bi-ideals $B(S)$ of S. In fact $B(S)$ forms a subsemigroup. We prove that S is clifford if and only if $B(S)$ is $\mathcal{H}$-commutative. Furthermore we give necessary and sufficient condition for $B(S)$ to be band.

5 Plan of Work:

I want to investigate the structure of ordered semigroups as follows:

(1) Characterizing the set of all bi-ideals of an ordered semigroup.
(2) Characterizing the set of all minimal bi-ideals of an ordered semigroup.
(3) Representing grouplike ordered semigroups as a union of minimal bi-ideals of an ordered semigroup.

6 Work Done:

In paper (1) we have introduced the notions of group like ordered semigroups, completely simple ordered semigroups and Clifford(left Clifford) ordered semigroups and then we have shown that every completely regular ordered semigroup is union of group(left group) like ordered semigroups. Later we have shown that the Green's relation $\mathcal{J}$ is the least complete semilattice congruence on an completely regular ordered semigroup. Using this result we finally get that an ordered semigroup S is completely regular if and only if it is a complete semilattice of completely simple ordered semigroups. In influence to Cao's remark in [6], we have characterized the structural representation of Clifford and left Clifford ordered semigroups by showing that an ordered semigroup is Clifford ordered semigroup if and only if it is a complete semilattice of group like ordered semigroups, where as an ordered semigroup is left Clifford ordered semigroup if and only if it is a complete semilattice of left group like ordered semigroups.

As another characterization of completely regular ordered semigroups we have discussed about the nil extensions of the same in the paper(2). Also we have characterized the semigroups which are nil extensions of group like ordered semigroups and Clifford ordered semigroups.

In paper(3) we have studied the behavior of radicals of Green's relations on the completely regular ordered semigroups.

References

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